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ABSTRACT

All have benefited from the industrial revolution which began in the 18th century and saw the gradual replacement of manual labor by machine. In the late 20th century, another revolution has been brought about by the rapid development of increasingly cheap and powerful computer-based technologies. Whereas the industrial revolution occurred through the mechanization of manual labor, the current electronic revolution is being achieved through a mechanization of certain sorts of intellectual skills. The brain rather than the hand is being made redundant. Initially from the mathematical point of view, the sort of intellectual skills that could be mechanized on an everyday basis might be dismissed as being relatively elementary in nature. However, high- level mathematical skills which used to be solved by complex algebraic expressions are now solved by handheld computer algebra systems. The key then is to see that emerging technologies do appear to threaten to make much of what was taught in the past redundant but they also offer real opportunities to enhance the mathematical capabilities of students. (Author/MVL)

# Educating students for their future not our past: a challenge for teachers of mathematics[4]



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We have all benefitted from the industrial revolution which began in the 18th century and which saw the gradual replacement of manual labour by machine. In the late twentieth century, we are going through another revolution brought about by the rapid development of increasingly cheap and powerful computer based technologies. Whereas the industrial revolution occurred through the mechanisation of manual labour, the current electronic revolution is being achieved through a mechanisation of certain sorts of *intellectual* skills. As stated by Atiyah (1986, p. 43), 'It is the brain rather than the hand that is being made redundant'.

Initially, from the mathematical point of view, the sort of intellectual skills that could be mechanised on an everyday basis might be dismissed as being relatively elementary, in nature. For example, the performance of routine arithmetic operations through the use of adding machines and more recently handheld calculators. However, increasingly, we are entering a world in which many more of what were previously considered to be high level mathematical skills, for example the ability to manipulate complex algebraic expressions, are now also capable of being mechanised on an everyday basis through the emergence of handheld computer algebra systems (CAS).

How should we as mathematics teachers react to this ever increasing mechanization of intellectual skills, the teaching of which is our 'bread and butter'? One approach is to do as the Luddites did, destroy the machines: in modern terms, ban the technology from the classroom as is currently happening in California (see for example, Becker and Jacob, 1998) and, in the process, continue to prepare students for the world of our past. The other approach is to realize that, while the emerging technologies do appear to threaten to make much of what we taught in the past redundant, they also offer real opportunities to enhance the mathematical capabilities of our students (see for example, Ralston, 1999). They have done so in the past and they will continue to do so in the future. What is so difficult for most of us at present is both the pace and the scope of the change. For many of us, it is almost equivalent to experiencing a 'thousand' years of change in a professional life time, as we can see by tracing technological developments in the mathematics classroom over the past 30 or so year.

[4] An earlier version of this paper was presented at ICME 5 (Jones, 1996)

**Looking back :**

If a 11 year old student in the 1960's was asked to evaluate 234 □346, they would have most likely reached for a pencil and paper, written the two numbers down on the paper, one above the other, and then performed the process of a long multiplication algorithm in a similar manner to that shown below:

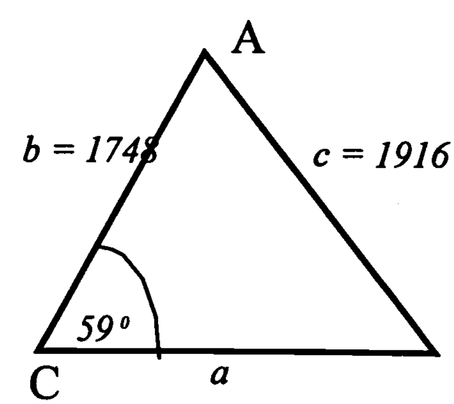
$$\begin{array}{r} 234 \\ \underline{346} \\ 1404 \\ 936\phantom{0} \\ \underline{702\phantom{00}} \\ \underline{80964} \end{array}$$

This is not a technology free process. In carrying out this computation, the student has used a technology, pencil and paper, to record the numbers to be multiplied. This in turn frees her mind to perform a series of mental operations on these numbers, the results of which are recorded sequentially for subsequent processing. Few students of that era were trained to carry out such computations without relying on such technology. On moving into secondary school, our student was required to move beyond whole number arithmetic and learn how to operate on numbers involving decimals. To assist her in that task she would have been introduced to a new technology, the table of logarithms. Tables of logarithms enabled complex arithmetic expressions involving decimal numbers to be transformed into less complex sums, differences or simple multiples, which could be systematically evaluated with the aid of pencil and paper as shown below.

| number | logarithm |
|---|---|
| 2.34 | 0.3692 |
| 0.0346 | $\bar{1}.4609$ |
| 0.08096 | $\bar{1}.0917$ |

Up until the 1970's, the table of logarithms was the only computational technology routinely available to students in the classroom and its role was pivotal in certain areas of the mathematics curriculum, as can be seen from the worked example of an application of the sine rule taken from a 1960's mathematics textbook (Rose, 1964).

Example 14. - Solve the ΔABC completely when $c$ = 1916 ft., $b$ = 1748 ft. and C = 59°.



To find B – $\frac{\sin B}{b} = \frac{\sin C}{c}$

and hence – $\sin B = \frac{b \sin C}{c}$

Taking logs throughout –

$$\begin{aligned}
\log \sin B &= \log 1748 + \log \sin 59^\circ - \log 1916 \\
&= 3.2425 + \bar{1}.9331 - 3.2823 \\
&= \bar{1}.8933 = \log \sin 51^\circ 28' \\
B &= \underline{51^\circ 28'} \\
\text{Then} - \quad A &= 180^\circ - (59^\circ + 51^\circ 28') \\
&= \underline{69^\circ 32'}
\end{aligned}$$

This example clearly illustrates the pivotal role played by logarithmic computations in solving such problems. Without these computational skills, such problems could not be solved, at least in the classroom. Unfortunately, for many teachers, the need to acquire such skills became an end in itself, rather than a means to an end, only made necessary by the lack of a practical alternative. Once the electronic calculator became common place in the classroom, the need for tables of logarithm for computations became unnecessary. Yet, at a workshop conducted for teachers on the use of the first electronic scientific calculators in the early seventies (Barling, 1995), one of the reasons given to teachers for introducing the calculator into their classroom was that it would obviate the need for students to use logarithm tables and would considerably speed up the process. It was stated that the calculator could be used to generate the logarithms, do the additions and then take the antilogarithms to obtain the required answer!

Why do such things happen? In part, it is due to the general lack of recognition that mathematics, like all human intellectual activity is always shaped by the available technology, but that, with time, the technologies 'become so deeply a part of our consciousness that we do not notice them' (Pea, 1993, p. 53). As a result, the technology effectively becomes 'invisible', while the activities it generates can come to be seen as mathematical activities in their own right, for example, carrying out calculations using logarithms. Hence, when a new technology such as the electronic calculator is introduced, it is common for it to be promoted as a means of 'enhancing' the teaching of such activities, even though the technology itself has been designed to obviate the need for such calculations. Kaput (1992, p.548) has termed this phenomenon 'retrofitting'. The irony of using a technology such as a calculator to help complete computations with logarithms should not be lost on anybody. Yet today, with CAS equipped graphics calculators (for example, the TI-89) now beginning to find their place in the everyday mathematics classroom in a number of countries, the suggestion that CAS might be used to help improve students' pencil and paper algebraic manipulative skills has a similar ring.

The tendency for aspects of mathematical practice to becomes an end in themselves rather than a means to an end, is clearly illustrated in modern calculus texts. For example, let us say that we wish to calculate the length of the arc of the curve $y = \ln x$

between $x = 1$ and $x = \sqrt{3}$. The solution reproduced below is based on a solution given in a typical calculus text (Grossman, 1977).

SOLUTION. Here $f'(x) = \frac{1}{x}$ so that

$$s = \int_1^{\sqrt{3}} \sqrt{1 + \frac{1}{x^2}}\,dx = \int_1^{\sqrt{3}} \frac{\sqrt{x^2+1}}{x}\,dx$$

Let $u = \tan\theta$ so that

$$s = \int_{\pi/4}^{\pi/3} \frac{\sec\theta\sec^2\theta}{\tan\theta}\,d\theta = \int_{\pi/4}^{\pi/3} \frac{\sec\theta(1+\tan^2\theta)}{\tan\theta}\,d\theta$$

$$= \int_{\pi/4}^{\pi/3}\left(\frac{\sec\theta}{\tan\theta} + \sec\theta\tan\theta\right)d\theta = \int_{\pi/4}^{\pi/3}\left(\frac{1}{\cos\theta}\frac{\cos\theta}{\sin\theta} + \sec\theta\tan\theta\right)d\theta$$

$$= \int_{\pi/4}^{\pi/3}(\csc\theta + \sec\theta\tan\theta)\,d\theta$$

$$= \left(\ln\left|\csc\theta + \cot\theta\right| + \sec\theta\right)\Big|_{\pi/4}^{\pi/3}$$

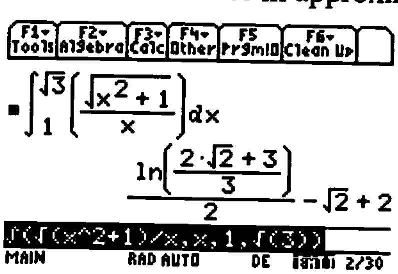
$$= \left\{\left[-\ln\left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) + 2\right] - \left[-\ln\left(\sqrt{2}+1\right) + \sqrt{2}\right]\right\}$$

$$= \ln\left(\sqrt{2}+1\right) - \ln\sqrt{3} + 2 - \sqrt{2} = \ln\left(\frac{\sqrt{2}+1}{\sqrt{3}}\right) + 2 - \sqrt{2}$$

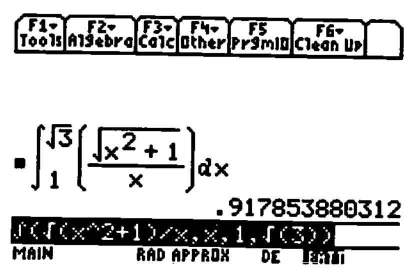
In looking at this solution we see that it bears an uncanny similarity to the 1960's textbook solution to the sine rule problem. First, some theoretical knowledge is used to set up the solution to the problem with pencil and paper. In the case of the sine rule application this results in a complex arithmetic expression which is then evaluated with the aid of log tables. In the arc length problem the solution is set up in the form of a definite integral which is then evaluated using an appropriate substitution and some algebraic manipulation to enable the original integral to be transformed into standard form. The results of the manipulation are recorded with pencil and paper and presumably a table of standard integrals is used in the end to help evaluate the resulting integrals. From figure 2 we can see that the arc length is

$$\ln\left(\frac{\sqrt{2}+1}{\sqrt{3}}\right) + 2 - \sqrt{2} = 0.9178538803.$$

However, if we have access to a graphics calculator with symbolic processing capabilities like the TI-89, we can simply utilize the integration facility to obtain the answer in exact form or in approximate numerical form, see figure 1.




Exact form




Approximate form

Figure 1: Using a TI-89 to evaluate the integral to obtain the arc length in both exact form and approximate form.

For most of us who learnt calculus as a pencil and paper based activity, it would be hard to accept that the steps involved in evaluating the definite integral in the arc length problem are not worthwhile mathematics, yet, if the true purpose of the activity was to evaluate the arc length, then the process as a whole may have no more intellectual value to the majority of students than the mastering of the skills needed to carry out complex arithmetic computations with tables of logarithms. Just as the electronic calculator was designed to avoid the need for human beings to carry out complex arithmetic computations by hand, a graphics calculator with numerical integration capabilities is designed to avoid the need for human beings to, amongst other things, evaluate complex definite integrals. This is challenging to those of us for whom the only technology supporting our calculus activities was pencil and paper and possibly tables of standard integrals. We had to master integration methods to solve more advanced problems, just as students in the past had to master computations with logarithms to solve more advanced mathematical problems. Thus we see that the available technology is a prime determinant of what mathematics we do in the classroom and how we do it, both now and in the past. So what is different now?

**Intelligent technology:**

To explore this question at a level that enables us to go beyond the specific technology involved, we need to recognise that the technologies we have used to support the teaching and learning of mathematics in the classroom, both now and in the past, can be regarded as 'intelligent' in the sense that they can 'undertake significant cognitive processing on behalf of the user' (Salamon, Perkins & Globerson, 1991. p. 4). Even pencil and paper, when used to support mathematical activity, can be regarded as intelligent technology. For example, when carrying out algebraic manipulations using pencil and paper, we record the results of intermediate steps so that we do not have to keep these results in our working memory at the same time as we carry out the mental processes involved in the manipulation. Thus pencil and paper can be regarded as being intelligent in that we use it to share the cognitive load when carrying out algebraic manipulations. Similarly, the use of a table of standard integrals shares the cognitive load of evaluating a complex integral by reducing the information we need to keep in working memory or retrieve from long term memory whilst carrying out the intermediate steps in the process.

If the older pencil and paper based technologies can be regarded as intelligent, what then differentiates them from the newer computer based intelligent technologies? Whereas the older technologies can share the cognitive load by acting as storage devices, computer based technologies not only store information but also have the added dimension of being able to carry out significant processing of that information with minimal intellectual input from the user. For example, a graphics calculator with symbolic processing capabilities can store an algebraic expression but then, on command, carry out a variety of algebraic processes of the sort that would have required considerable mental effort on our behalf when working with pencil and paper only. The ability of computer based technology to both store and process mathematical information significantly increases the potential to share the intellectual burden with the user. However, computer based technology cannot plan, model, synthesise, interpret, etc. At present, these are intellectual abilities possessed only by burden with the user. However, computer based technology cannot plan, model, synthesise, interpret, etc. At present, these are intellectual abilities possessed only by the human mind, which can, of course, also store information and carry out rule based processing. A schematic view of the differing intellectual capabilities of pencil and paper based technology, computer based technology and the human mind is shown in figure 2. paper based technology

| |
|---|
| information storage |

computer based technology

| | |
|---|---|
| information storage | rule based processing |

human 'technology'

| | | |
|---|---|---|
| information storage | rule based processing | modelling<br>interpretation<br>etc |

Figure 2:Schematic overview of the differing intellectual capabilities of pencil-and-paper based technology, computer based technology and the human mind

The higher level thinking skills are the skills that we ultimately value in mathematics but, in practice, we spend most of the time teaching and developing processing skills, particularly algebraic manipulative skills. In part this is because, in a pencil and paper based classroom, mastery of these skills is a necessary prerequisite to using mathematics at a higher intellectual level. Unfortunately, because of the time taken and the intellectual effort needed to develop such skills, the greater part of classroom instruction has been devoted to the acquisition of these skills. As a result, the mastery of these skills has become the primary goal of the majority of mathematics classrooms, and mastery of these skills has become equated with mathematical ability. Again, the means have become the end. Thus any technology that appears to enable a person to carry out such tasks at the push of a button challenges our traditional concept of what constitutes mathematical ability. However, this is only a problem if we continue to view mathematical intelligence as residing entirely within the individual. As we will see, it also limits our thinking about the potential educative role of technology in mathematics.

## Intelligent partnership and mathematical ability:

What are the educational consequences of thinking of the technology we use to support mathematical as being 'intelligent'? One is the potential for the development of what has been termed an *intelligent partnership* . In an intelligent partnership, the potential exists for intellectual performance of the partnership to be 'far more "intelligent" than the human alone' (Salamon et al, 1991, p. 4). For example, with access to technology such as a graphics calculator, students have the potential to pursue graphical methods of solution and analysis that greatly exceed what they could ever hope to achieve with a pencil and paper alone, even in principle.

This possibility of students forming intelligent partnerships with technology in mathematics gives them the potential to work at a level in mathematics that may be totally unachievable without the technology. This, in effect, calls into question our traditional notions about what constitutes mathematical intelligence and how it should be assessed. Should it be measured by the mathematical performance of the student working without any technological aid, or does the possibility now arise of it being also recognised as the mathematical performance of a joint system? If we accept that a student working in an intelligent partnership with computer based technology is a legitimate and valued form of mathematical activity, then we must consider the possibility that appropriate assessment of mathematical intelligence involves assessment of that partnership. Further, given that, in the long run, almost all real mathematical activity involves the use of some supportive computer based technology, it could be argued that one of our prime pedagogic interests in mathematics should be directed at the task of developing instructional strategies for building and assessing the mathematical intelligence of such partnerships and not just the individual working alone.

Unfortunately, intelligent partnerships do not appear to be self generating and the challenge for teachers is to develop instructional strategies that promote their formation. And, more importantly, it is unlikely that they will be realised unless students have the same sort of access to the necessary technology as they currently have to pencil and paper. In this regard, handheld technology such as the graphics calculator is likely to have far greater potential than a computer as it is cheap enough and small enough to be in the hands of students at all times. Finally, there is also a need to reassess what is taught, as the knowledge and understandings needed to develop an intelligent mathematical partnership when working with technology are almost certain to differ in some significant ways from those needed for students who will do all their mathematics without access to technology.

**Summary and conclusion:**

In this paper I have argued that when thinking about the role of the newer hand held computer based technologies in the mathematics classroom we first need to realise that we have always used technology to support mathematical activity in the classroom but, because of its familiarity we have not been very good at separating out what is mathematics in its own right and what is only of value because of the technology we have at our disposal. As a consequence, whenever a new and different technology emerges there has been a natural tendency to retrofit the new technologies to the mathematics activities with which we have become most familiar without any real regard for their relevance in the new technological environment. While this retrofitting of the technology has superficially appeared to bring about significant pedagogic gains in that it enhanced the learning of skills previously difficult to teach, such uses of the new technologies activities are more often than not of transitional value (see, also, Kaput, 1992, p. 517). Secondly, we need to recognize that the technologies we have used to support the teaching and learning of mathematics in the classroom, both now and in the past, can be regarded as 'intelligent' in that they have the ability to reduce cognitive load. However, the new computer based technologies are qualitatively different from the older pencil and paper based technologies because of their ability to both store and process mathematical information. Finally, in recognizing the 'intelligent' nature of the technology we open up the potential for the formation of intellectual partnerships which have the potential to be far more mathematically intelligent than human intelligence alone. This, in effect was what we were aiming for when the technology of the mathematics classroom was pencil and paper based, but we failed to recognize this because the intellectual potential of these technologies was far less obvious than that of the newer technologies. From this point of view, the goals of mathematics education are not under challenge. What is under challenge is the means by which we try to achieve these goals.

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